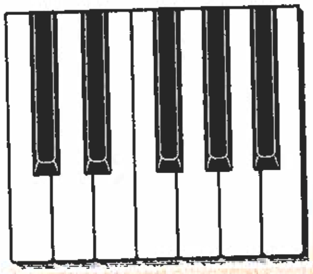
WRITING AND SIMPLIFYING RATIOS



A **ratio** is a comparison of two like quantities by division. In the diagram at the left, the ratio of white keys to black keys is 7 to 5. This ratio can also be expressed with a colon, 7:5, or as a fraction, $\frac{7}{5}$.

The order in which the ratio is written is important. Note that the ratio of 5 black keys to 7 white keys is written 5 to 7, 5:7, or $\frac{5}{7}$.

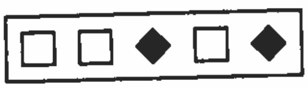
Express the ratio of black keys to total keys in three ways.

There are 5 black keys and 12 keys in all. The ratio of black keys to total keys can be written:

5 to 12, 5:12, or $\frac{5}{12}$.

EXAMPLE A

Use the figures below to write the ratio of ◆s to □s.



There are 2 ◆s and 3 □s.

The ratio of ◆s to □s is 2 to 3, 2:3, or $\frac{2}{3}$.

When the numbers compared in a ratio have a common factor, you can write the ratio as an equivalent ratio in simplest form. This is shown in Example B.

EXAMPLE B

At a meeting of 54 people, there are 12 left-handed people. Write the ratio of the number of left-handed people to the total number of people.

$\frac{12}{54}$ ← Write the ratio of the number of left-handed people to the total number of people.

$\frac{12}{54} = \frac{2}{9}$ ← Simplify the fraction.

The ratio of the number of left-handed people to the total number of people is 2 to 9, 2:9, or $\frac{2}{9}$.

Both terms of a ratio must have the same unit. If the quantities have different units of measure, they can be compared as a ratio, provided it is possible to relate one unit to the other. For example, inches and feet are different units of length, but they are related by the fact 12 in. = 1 ft. We use a fact such as 12 in. = 1 ft to write a conversion factor. By using an appropriate conversion factor, we can cancel the units of measure and get a ratio that has only numbers. In Example C, the conversion factor is $\frac{12 \text{ in.}}{1 \text{ ft}}$.

EXAMPLE C

What is the ratio of 2 feet to 6 inches in simplest form?

$\frac{2 \text{ ft}}{6 \text{ in.}} = \frac{2 \text{ ft}}{6 \text{ in.}} \times \frac{12 \text{ in.}}{1 \text{ ft}}$ ← Write the ratio; multiply by the conversion factor.

$= \frac{2 \cancel{\text{ft}}}{6 \cancel{\text{in.}}} \times \frac{12 \cancel{\text{in.}}}{1 \cancel{\text{ft}}}$ ← Cancel the units of measure.

$= \frac{24}{6}$ ← Multiply.

$= \frac{4}{1}$ ← Simplify.

The ratio of 2 feet to 6 inches is 4 to 1, 4:1 or $\frac{4}{1}$.

It is impossible to express two quantities as ratios if the terms have unlike units of measure that cannot be represented as like units. For example, inches and pounds cannot be compared as ratios.

EXAMPLE D

What is the ratio of 3 hours to 15 minutes in simplest form?

Use the conversion factor $\frac{60 \text{ min}}{1 \text{ hr}}$ to cancel the units of measure in the ratio.

$\frac{3 \text{ hr}}{15 \text{ min}} = \frac{3 \text{ hr}}{15 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}}$ ← Write the ratio; multiply by the conversion factor.

$= \frac{3 \cancel{\text{hr}}}{15 \cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{1 \cancel{\text{hr}}}$ ← Cancel the units.

$= \frac{180}{15}$ ← Multiply.

$= \frac{12}{1}$ ← Simplify.

The ratio of 3 hours to 15 minutes is 12 to 1, 12:1, or $\frac{12}{1}$.

When an object is represented in a drawing or in a model, a scale is used to show the dimensions of the object proportionally. A **scale** is a ratio that compares the dimensions in the drawing or the model to the actual dimensions. A scale is given as a ratio with the names of the units included. This is shown in Example E.

EXAMPLE E

A distance of 90 miles is represented by 3 inches on a map. Write the scale of the map in simplest form.

$\frac{3 \text{ in.}}{90 \text{ mi}}$ ← Write the ratio.

$\frac{3 \text{ in.}}{90 \text{ mi}} = \frac{3 \text{ in.} \div 3}{90 \text{ mi} \div 3} = \frac{1 \text{ in.}}{30 \text{ mi}}$ ← Divide to get a numerator of 1; keep the units in the fraction.

1 in.:30 mi ← A scale may be written as a ratio using a colon.

The scale of the map is 1 in.:30 mi.

EXAMPLE F

A wall 20 feet long is represented by an 8-inch segment on a blueprint. What is the scale of the blueprint, written as an equation?

$\frac{8 \text{ in.}}{20 \text{ ft}}$ ← Write the ratio.

$\frac{8 \text{ in.}}{20 \text{ ft}} = \frac{8 \text{ in.} \div 8}{20 \text{ ft} \div 8} = \frac{1 \text{ in.}}{2.5 \text{ ft}}$ ← Divide to get a numerator of 1; keep the units in the fraction.

1 in. = 2.5 ft ← A scale is sometimes written as an equation. This scale indicates that 1 inch on the blueprint represents an actual length of 2.5 feet.

The scale of the blueprint is given by the equation 1 in. = 2.5 ft.

EXERCISES 11-1

Express each of the following ratios in three ways. Write each ratio in simplest form.

1. Of 48 vehicles sold last month at a dealership, 16 were hybrids. What is the ratio of the number of hybrids sold to the total number of cars sold?
2. What is the ratio of 60 volts to 8 volts?
3. A transformer has 1,128 primary turns and 16,848 secondary turns. Express the ratio of primary turns to secondary turns in lowest terms.
4. A contractor spent 90 minutes preparing an estimate for a job and 24 hours doing the job. What is the ratio of the time spent preparing the estimate to the time spent doing the work on the job?
5. What is the ratio of 35 millimeters to 35 meters?
6. In a handful of coins, there are 4 nickels, 4 dimes, and 12 quarters. What is the ratio of the number of dimes to the total number of coins?

Write each scale in simplest form and as an equation.

7. Two cities that are actually 350 kilometers apart are 7 centimeters apart on a map. What is the scale of the map?
8. A drawing of a solar heating panel on a blueprint is 2 centimeters wide. The actual panel is 3 meters wide. What is the scale of the drawing?
9. The length of a room on a drawing is 3.25 inches. The actual room will be 15 feet $8\frac{1}{2}$ inches. What is the scale of the drawing?
10. **Challenge:** What is the ratio in simplest form of 80 centimeters to 2 kilometers?

WRITING UNIT RATES

A **rate** is a type of ratio that compares two unlike quantities. For example, the rate at which cars are passing through a tollbooth could be expressed as 50 cars per 30-minute interval. In this case, the unlike quantities are cars and minutes.

A **unit rate** is a rate in which the denominator is 1 unit. A speed limit of 65 on a U.S. interstate means that the drivers should not be driving any faster than 65 miles per hour. The unit rate 65 miles per hour can be written $\frac{65 \text{ mi}}{1 \text{ hr}}$. Note that the denominator is 1 hour.

Express the ratio $\frac{330 \text{ mi}}{6 \text{ hr}}$ as a unit rate.

$\frac{330 \text{ mi}}{6 \text{ hr}} = \frac{330 \text{ mi} \div 6}{6 \text{ hr} \div 6}$ ← Write the ratio. Divide both parts of the ratio by the numerical part of the denominator.

$= \frac{55 \text{ mi}}{1 \text{ hr}}$ ← Simplify.

The unit rate is $\frac{55 \text{ mi}}{1 \text{ hr}}$, or 55 miles per hour.

EXAMPLE A

Express the rate $\frac{\$8.75}{2.5\text{ lb}}$ as a unit rate.

$$\frac{\$8.75}{2.5\text{ lb}} = \frac{\$8.75 \div 2.5}{2.5\text{ lb} \div 2.5}$$ ← Write the rate. Divide both parts of the rate by the numerical part of the denominator.

$$= \frac{\$3.50}{1\text{ lb}}$$ ← Simplify.

The unit rate is $\frac{\$3.50}{1\text{ lb}}$, or \$3.50 per pound.

EXAMPLE B

A case of 12 quarts of oil costs \$10.20. What is the unit cost of the oil?

$$\frac{\$10.20}{12\text{ qt}} = \frac{\$10.20 \div 12}{12\text{ qt} \div 12}$$ ← Write the rate. Divide both parts of the rate by the numerical part of the denominator.

$$= \frac{\$0.85}{1\text{ qt}}$$ ← Simplify.

The unit cost is $\frac{\$0.85}{1\text{ qt}}$, or \$0.85 per quart.

Language Box

For the cost of an item, unit rate is called **unit cost.**

EXAMPLE C

A tourist received 43.80 British pounds in exchange for \$60 in U.S. money. How many pounds did he receive for each U.S. dollar?

$$\frac{43.80\text{ pounds}}{\$60} = \frac{43.80\text{ pounds} \div 60}{\$60 \div 60}$$ ← Write the rate. Divide both parts of the rate by the numerical part of the denominator.

$$= \frac{0.73\text{ pounds}}{\$1}$$ ← Simplify.

He received 0.73 British pound for each U.S. dollar.

EXERCISES 11-2

Express each of the following rates as a unit rate.

1. $\frac{576\text{ mi}}{18\text{ gal}}$
2. $\frac{\$4.56}{12\text{ oz}}$
3. $\frac{\$73.50}{6\text{ shirts}}$
4. $\frac{640\text{ km}}{8\text{ hr}}$
5. $\frac{5{,}000\text{ people}}{50\text{ mi}^2}$
6. $\frac{\$15.60}{24\text{ lightbulbs}}$
7. $\frac{54\text{ ft}^3}{2\text{ yd}^3}$
8. $\frac{15{,}4000\text{ ft}}{5\text{ min}}$

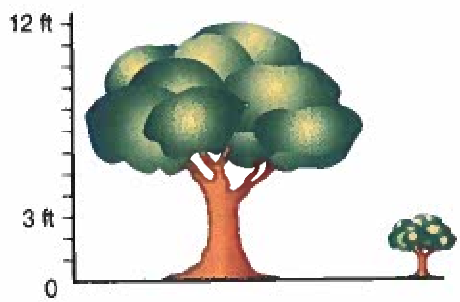
Helpful Hint

To find a unit rate, divide by the number of the quantity that follows the word *per*.

Answer each of the following.

9. If 70 feet of flat electrical cable can be purchased for \$164.50, what is the price per foot?
10. The resistance of a wire depends on its length. If a wire that is 16 feet long has a resistance of 0.4 Ω, what is the resistance per foot?
11. Paul received \$62 in Canadian dollars in exchange for \$50 in U.S. money. What was the exchange rate for each U.S. dollar?
12. Population density is the number of residents per square mile. In 2010, there were 75,000 residents living in a certain county with an area of 213 square miles. What was the population density of the county? Round your answer to the nearest whole number.
13. **Challenge:** Which product has the lower unit cost: product A at 48 ounces for \$3.84 or product B at 64 ounces for \$5.44? Explain your answer.

IDENTIFYING AND SOLVING PROPORTIONS



A **proportion** is an equation that states that two ratios are equal. Proportions can be written several ways. The proportion $\frac{1}{4} = \frac{3}{12}$ can also be written 1:4 = 3:12. It is read *1 is to 4 as 3 is to 12.*

In the proportion $a:b = c:d$, the terms a and d are the **extremes,** and the terms b and c are the **means.** This proportion could also be written $\frac{a}{b} = \frac{c}{d}$.

means

$$a \div b = c \div d$$

extremes

When one term in a proportion is not known, a variable may be used in its place. For example, in the proportion $\frac{3}{8} = \frac{x}{40}$, the value of the term represented by x is unknown. Cross multiplication can be used to find the value of x. Do this by setting the product of the means equal to the product of the extremes and solving the equation for the variable.

$$\frac{3}{8} = \frac{x}{40}$$

Product of means	Product of extremes
$8x$	$3 \cdot 40$

The product of the means and the product of the extremes are called simply the *cross products.*

Language Box

Finding the value of a variable in a proportion is sometimes called *solving the proportion*.

Solve the proportion $\frac{3}{8} = \frac{x}{40}$.

Solve:

$\frac{3}{8} = \frac{x}{40}$ ← Write the proportion.

$8x = 3 \cdot 40$ ← Set the product of the means equal to the product of the extremes.

$8x = 120$

$\frac{8x}{8} = \frac{120}{8}$ ← Divide by 8 to isolate the variable.

$x = 15$

Check:

$\frac{3}{8} = \frac{x}{40}$ ← Write the original proportion.

$\frac{3}{8} \stackrel{?}{=} \frac{15}{40}$ ← Substitute your solution for x.

$8 \cdot 15 \stackrel{?}{=} 3 \cdot 40$

$120 = 120$ ✓ ← Verify that the cross products are equal.

$x = 15$

Helpful Hint

When a proportion is correct, the cross products are equal.

EXAMPLE A

Solve the proportion $\frac{30}{x} = \frac{120}{100}$.

Solve:

$\frac{30}{x} = \frac{120}{100}$ ← Write the proportion.

$120x = 30 \cdot 100$ ← Set the product of the means equal to the product of the extremes.

$120x = 3{,}000$

$\frac{120x}{120} = \frac{3{,}000}{120}$ ← Divide by 120 to isolate the variable.

$x = 25$

Check:

$\frac{30}{x} = \frac{120}{100}$ ← Write the original proportion.

$\frac{30}{25} \stackrel{?}{=} \frac{120}{100}$ ← Substitute your solution for x.

$\frac{6}{5} = \frac{6}{5}$ ✓ ← Verify that the ratios are equal.

$x = 25$

Helpful Hint

When a proportion is true, both ratios of the proportion are the same when written in simplest form.

EXAMPLE B

Solve the proportion $\frac{n}{0.6} = \frac{5}{3}$.

Solve:

$\frac{n}{0.6} = \frac{5}{3}$ ← Write the proportion.

$3n = 0.6 \cdot 5$ ← Set the product of the extremes equal to the product of the means.

$3n = 3$ ← Solve the equation for n.

$n = 1$

Check:

$\frac{n}{0.6} = \frac{5}{3}$ ← Write the original proportion.

$\frac{1}{0.6} \stackrel{?}{=} \frac{5}{3}$ ← Substitute your solution for n.

$1 \cdot 3 \stackrel{?}{=} 0.6 \cdot 5$

$3 = 3$ ✓ ← Verify that the cross products are equal.

$n = 1$

EXAMPLE C

Solve the proportion $\frac{25}{10a} = \frac{20}{36}$.

Solve:

$\frac{25}{10a} = \frac{20}{36}$ ← Write the proportion.

$20 \cdot 10a = 25 \cdot 36$ ← Set the product of the means equal to the product of the extremes.

$200a = 900$

$\frac{200a}{200} = \frac{900}{200}$ ← Divide both sides of the equation by 200.

$a = 4.5$

Check:

$\frac{25}{10a} = \frac{20}{36}$ ← Write the original proportion.

$\frac{25}{10(4.5)} \stackrel{?}{=} \frac{20}{36}$ ← Substitute your solution for a.

$\frac{25}{45} \stackrel{?}{=} \frac{20}{36}$

$\frac{5}{9} = \frac{5}{9}$ ✓ ← Verify that the ratios are equal.

$a = 4.5$

A variable may appear in more than one term in a proportion. In that case, write the proportion and cross multiply as you would with any proportion. Then use the steps you have learned for solving equations to isolate the variable and find the value of the variable. This is shown in Example D.

EXAMPLE D

Solve the proportion $\frac{5x-3}{4} = \frac{5x+3}{6}$.

Solve:

$\frac{5x-3}{4} = \frac{5x+3}{6}$ ← Write the proportion.

$6(5x-3) = 4(5x+3)$ ← Set the product of the extremes equal to the product of the means.

$30x - 18 = 20x + 12$ ← Use the Distributive Property to remove parentheses.

$10x - 18 = 12$ ← Subtract $20x$ from both sides.

$10x = 30$ ← Add 18 to both sides.

$\frac{10x}{10} = \frac{30}{10}$ ← Divide by 10 to isolate the variable.

$x = 3$

Check:

$\frac{5x-3}{4} = \frac{5x+3}{6}$ ← Write the original proportion.

$\frac{5(3)-3}{4} \stackrel{?}{=} \frac{5(3)+3}{6}$ ← Substitute your solution for x.

$\frac{15-3}{4} \stackrel{?}{=} \frac{15+3}{6}$

$\frac{12}{4} \stackrel{?}{=} \frac{18}{6}$

$3 = 3$ ✓ ← Verify that the ratios are equal.

$x = 3$

EXERCISES 11-3

Solve each proportion.

1. $\frac{5}{4} = \frac{75}{n}$
2. $\frac{a}{36} = \frac{1}{9}$
3. $\frac{2}{3} = \frac{8}{2x}$
4. $\frac{90}{64} = \frac{n}{32}$
5. $\frac{4w-9}{5} = \frac{w+3}{3}$
6. $\frac{3a}{7} = \frac{6}{1}$
7. $\frac{x}{35} = \frac{16}{40}$
8. $\frac{6x-2}{7} = \frac{5x+7}{8}$
9. $\frac{352}{11} = \frac{w}{0.4}$
10. $\frac{3}{14} = \frac{3n}{126}$
11. $\frac{11}{n} = \frac{77}{14}$
12. $\frac{6}{5w} = \frac{4.8}{9}$
13. Six is to 15 as some number x is to 45. Set up a proportion and solve for x.
14. The ratio of 10 to 0.5 is the same as the ratio of 40 to some number x. What is the value of x?

15. The resistance of a wire is proportional to its length. If a 24-foot wire has a resistance of 0.6 Ω, what is the resistance of 42 feet of the same wire?

16. The ratio of the length to the width of a printed circuit board is 4.5:3.2. The maximum width of a circuit board is 406.4 millimeters. What is the maximum length?

17. **Challenge:** If you multiply both sides of the proportion $\frac{a}{b} = \frac{c}{d}$ by bd, what equation do you get? What does the equation represent?

SOLVING DIRECT PROPORTION PROBLEMS

Servings	Oats	Water
2	1 cup	2 cups
4	2 cups	4 cups
8	4 cups	8 cups

Two variables, x and y, are **directly proportional** if the ratio of y to x is a nonzero constant. This can be expressed by the equation $\frac{y}{x} = k$, where k is a nonzero number. For example, the ratio of y to x may be 4 to 8. In this case, $\frac{y}{x} = \frac{4}{8}$, and the constant ratio in simplest form is $\frac{1}{2}$. To set up a direct proportion, compare the quantities in the same way in each ratio of the proportion. Consider the amounts of oats and water for various servings of oatmeal shown at the left.

Language Box

In the equation $\frac{y}{x} = k$ describing a direct proportion, k is called the **constant of variation** or *constant of proportionality.*

For two servings of oatmeal, the ratio of oats to water is $\frac{\text{1 cup oats}}{\text{2 cups water}}$, or 1 to 2. For four servings, the ratio of oats to water is $\frac{\text{2 cups oats}}{\text{4 cups water}}$, or 1 to 2. The same ratio holds true for eight servings: $\frac{\text{4 cups oats}}{\text{8 cups water}}$, or 1 to 2. We say that the amount of water used is directly proportional to the amount of oats used. That is, as one increases, so does the other.

How many cups of water are used with 3 cups of oats to make the same type of oatmeal?

Let x represent the number of cups of water needed. Solve the proportion $\frac{\text{3 cups oats}}{x\text{ cups water}}$ or $\frac{3}{x} = \frac{1}{2}$.

Solve:

$\frac{3}{x} = \frac{1}{2}$ ← Write the proportion.

$3 \cdot 2 = 1 \cdot x$ ← Cross multiply to solve the proportion.

$6 = x$

Check:

$\frac{3}{x} = \frac{1}{2}$ ← Write the original proportion.

$\frac{3}{6} \stackrel{?}{=} \frac{1}{2}$ ← Substitute your solution for x.

$\frac{1}{2} = \frac{1}{2}$ ✓ ← Verify that the ratios are equal.

For 3 cups of oats, 6 cups of water are needed.

EXAMPLE A

If 3 spools of wire of equal weight weigh a total of 54 pounds, how much would 7 of the same spools weigh?

Solve:

Let w represent the weight of the 7 spools.

$\frac{3 \text{ spools}}{54 \text{ pounds}} = \frac{7 \text{ spools}}{w \text{ pounds}}$ ← Compare the quantities in the same way in each ratio.

$\frac{3}{54} = \frac{7}{w}$ ← Write the proportion with only numbers and variables.

$3w = 7 \cdot 54$ ← Cross multiply and then solve the equation.

$\frac{3w}{3} = \frac{378}{3}$

$w = 126$

Check:

$\frac{3}{54} = \frac{7}{w}$ ← Write the original proportion.

$\frac{3}{54} \stackrel{?}{=} \frac{7}{126}$ ← Substitute your solution for w.

$\frac{1}{18} = \frac{1}{18}$ ✓ ← Verify that the ratios are equal.

Seven of the same spools of wire would weigh 126 pounds.

A unit rate may be given as part of a direct proportion. This is shown in Example B. Continue to check the solutions to the remaining examples in the lesson on your own.

EXAMPLE B

A certain car gets an average of 22.5 miles to a gallon of gas. How many gallons will the car use for a 900-mile trip?

Let n represent the number of gallons.

$\frac{22.5 \text{ miles}}{1 \text{ gallon}} = \frac{900 \text{ miles}}{n \text{ gallons}}$ ← Compare the quantities in the same way in each ratio.

$\frac{22.5}{1} = \frac{900}{n}$ ← Write the proportion with only numbers and variables.

$900 = 22.5n$ ← Cross multiply and then solve the equation.

$\frac{900}{22.5} = \frac{22.5n}{22.5}$

$40 = n$

The car will use 40 gallons of gas for a 900-mile trip.

Frequently, subscripts are used with variables to identify the terms in a proportion. If variables x and y are directly proportional, a subscript of 1 is used in the first ratio and a subscript of 2 is used in the second ratio. The direct proportion is written $\frac{y_1}{x_1} = \frac{y_2}{x_2}$. This is shown in Example C.

EXAMPLE C

Variables x and y are directly proportional. If $x_1 = 5$, $y_1 = 9$, and $x_2 = 15$, determine the value of y_2.

$\frac{y_1}{x_1} = \frac{y_2}{x_2}$ ← Set up the direct proportion.

$\frac{9}{5} = \frac{y_2}{15}$ ← Substitute the given values.

$5y_2 = 9 \cdot 15$ ← Cross multiply and then solve the equation.

$\frac{5y_2}{5} = \frac{135}{5}$

$y_2 = 27$

$y_2 = 27$

You may wonder if there is more than one way to set up a direct proportion for a set of numbers. The answer is yes. A proportion is true if the cross products are equal. For example, $\frac{2}{3} = \frac{6}{9}$ is true because $2 \cdot 9 = 6 \cdot 3 = 18$. For any true proportion, you can arrange terms to get another true proportion as long as the cross product remains the same. The following are some of the true proportions that are equivalent to $\frac{2}{3} = \frac{6}{9}$. Notice that they all have the same cross product, 18.

$\frac{2}{6} = \frac{3}{9}$ $\quad \frac{9}{3} = \frac{6}{2}$ $\quad \frac{6}{2} = \frac{9}{3}$

EXAMPLE D

Variables x and y are directly proportional. If $x_1 = 3$, $y_1 = 5$, and $y_2 = 20$, determine the value of x_2 using three different proportions.

METHOD 1	METHOD 2	METHOD 3
$\frac{x_1}{y_1} = \frac{x_2}{y_2}$	$\frac{x_1}{x_2} = \frac{y_1}{y_2}$	$\frac{y_2}{y_1} = \frac{x_2}{x_1}$
$\frac{3}{5} = \frac{x_2}{20}$	$\frac{3}{x_2} = \frac{5}{20}$	$\frac{20}{5} = \frac{x_2}{3}$
$5x_2 = 3(20)$	$5x_2 = 3(20)$	$5x_2 = 20(3)$
$5x_2 = 60$	$5x_2 = 60$	$5x_2 = 60$
$x_2 = 12$	$x_2 = 12$	$x_2 = 12$

Because the proportions are all equivalent, the solution, $x_2 = 12$, is the same in all three cases.

EXERCISES 11-4

Assume that the variables x and y are directly proportional. Determine the missing value in each case.

1. If $x_1 = 5$, $y_1 = 125$, and $y_2 = 25$, determine the value of x_2.
2. If $y_1 = 6$, $x_2 = 15$, and $y_2 = 18$, determine the value of x_1.
3. If $x_1 = 2$, $y_1 = 16$, and $x_2 = 3.5$, determine the value of y_2.
4. If $x_1 = 32$, $y_1 = 80$, and $y_2 = 45$, determine the value of x_2.

5. If $x_1 = 45$, $x_2 = 0.5$, and $y_2 = 2$, determine the value of y_1.
6. If $x_1 = \$11.00$, $y_1 = 2$, and $y_2 = 3$, determine the value of x_2.
7. If $y_1 = 560$, $x_2 = 5$, and $y_2 = 100$, determine the value of x_1.
8. If $x_1 = 1$, $y_1 = 120$, and $x_2 = 1.75$, determine the value of y_2.

Solve each of the following problems.

9. The resistance in a wire is directly proportional to the length of the wire. If 1,000 feet of #18 copper wire has a resistance of 6.5 ohms, what is the resistance of 1,200 feet of #18 copper wire?
10. When the resistance is constant, the current, *I*, varies directly as the voltage, *V*, varies.
    (a) Write the proportion for this problem.
    (b) If $I = 4.25$ A when $V = 110$ V, what is the current when the voltage is 60 V? Round your answer to the nearest hundredth.
11. George's car gets an average of 21 miles to a gallon of gas. At this rate, how many gallons will the car use on a 378-mile trip?
12. In an inductive-resistive circuit, the inductive reactance varies directly as the frequency changes. In a given circuit, a frequency of 2500 Hz produces an inductive reactance of 650.4 Ω. What frequency would produce an inductive reactance of 1740 Ω? Round your answer to the nearest tenth.
13. In a transformer, the voltage across the windings is directly proportional to the number of turns in the windings. If the primary windings have 100 turns and 60 volts across them, what is the voltage in the secondary windings with 150 turns?
14. **Challenge:** Using a copier, an office assistant reduces a document 8 inches wide using a 75% ratio. Then she enlarges the new document using a 150% ratio. What is the width of the final document?

SOLVING INVERSE PROPORTION PROBLEMS

rate (mph)	×	time (hr)	=	200 (miles)
20	×	10	=	200
25	×	8	=	200
50	×	4	=	200
80	×	2.5	=	200

When two variables x and y are **inversely proportional,** the product of x and y is a nonzero constant. This can be expressed by the equation $xy = k$, where k is a nonzero number. For example, the product xy may be 200. In this case, $xy = 200$ and the constant of variation k is 200.

Recall that if x and y are directly proportional, then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ and $x_1y_2 = x_2y_1$. Two variables x and y are inversely proportional if $\frac{x_1}{y_2} = \frac{x_2}{y_1}$ and $x_1y_1 = x_2y_2$.

The table at the left shows several different rates of speed and corresponding times that all result in the same distance traveled, 200 miles. Note that as the rate increases, the time decreases, but the distance is the same in every case. In this situation, we say that the time is inversely proportional to the rate. That is, as one increases, the other decreases, but the product rt (xy) equals 200 (k) in each case.

How long would it take a train to go 200 miles at an average rate of 40 miles per hour?

$rt = d$ ← Write the formula *rate* × *time* = *distance*.

$40t = 200$ ← Substitute 40 for *r* and 200 for *d*.

$\frac{40t}{40} = \frac{200}{40}$ ← Solve.

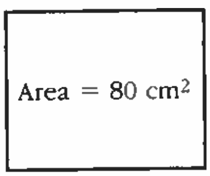
$t = 5$

It would take 5 hours to go 200 miles at 40 miles per hour.

Recall that the area of a rectangle is found using the formula area = length · width. For a given area, length is inversely proportional to width. This formula is used in Example A.

EXAMPLE A

Complete the table to show some different lengths and widths of rectangles that have an area of 80 cm².



Area	=	length	·	width	width
80 cm²	=	2 cm	·	____	40 cm
80 cm²	=	4 cm	·	____	20 cm
80 cm²	=	5 cm	·	____	16 cm
80 cm²	=	10 cm	·	____	8 cm
80 cm²	=	20 cm	·	____	4 cm

To solve an inverse proportion problem involving x_1, y_1, x_2, and y_2, you can begin with the equation $x_1y_1 = x_2y_2$.

EXAMPLE B

Variables *x* and *y* are inversely proportional. If $x_1 = 2$, $y_1 = 24$, and $x_2 = 16$, determine the value of y_2.

Language Box

The expressions *indirect proportion* and *inverse proportion* are both used to describe the relationship $x_1y_1 = x_2y_2$.

Solve:

$x_1y_1 = x_2y_2$ ← Write the equation.

$2 \cdot 24 = 16 \cdot y_2$ ← Substitute the given values.

$48 = 16y_2$ ← Solve the equation.

$\frac{48}{16} = \frac{16y_2}{16}$

$3 = y_2$

Check:

$2 \cdot 24 = 16 \cdot y_2$ ← Write the original equation.

$2 \cdot 24 \stackrel{?}{=} 16 \cdot 3$ ← Substitute your solution.

$48 = 48$ ✓ ← Verify that the products are equal.

$y_2 = 3$

EXAMPLE C

The capacitive reactance in a circuit varies inversely as the capacitance changes. A circuit with a capacitance 0.05 μF produces a capacitive reactance of 19 980 Ω. If the capacitance changes to 0.075 μF, find the new capacitive reactance.

Solve:

We will let $C_1 = 0.05$ μF, $X_{C1} = 19\,980$ Ω, and $C_2 = 0.075$ μF. We want to solve for X_{C2} in ohms.

$C_1X_{C1} = C_2X_{C2}$ ← Write the equation.

$(0.05)(19\,980) = 0.075X_{C2}$ ← Substitute the given values.

$999 = 0.075X_{C2}$ ← Solve the equation.

$$\frac{999}{0.075} = \frac{0.075X_{C2}}{0.075}$$

$13\,320 = X_{C2}$

Check.

$(0.05)(19\,980) = 0.075X_{C2}$ ← Write the original equation.

$999 \stackrel{?}{=} (0.075)(13\,320)$ ← Substitute your solution.

$999 = 999$ ✓ ← Verify that the products are equal.

The new capacitive reactance is 13 320 Ω.

EXAMPLE D

The time to do a certain job is indirectly proportional to the number of people working on the job. If 6 people can do the job in 90 hours, how many people are required to do the job in 30 hours?

Solve:

Let $x_1 = 6$, $y_1 = 90$, and $y_2 = 30$. Solve for x_2, the number of people required to do the job in 30 hours.

$x_1y_1 = x_2y_2$ ← Write the equation.

$6 \cdot 90 = x_2 \cdot 30$ ← Substitute the given values.

$540 = 30x_2$ ← Solve the equation.

$$\frac{540}{30} = \frac{30x_2}{30}$$

$18 = x_2$

So, 18 people are required to do the job in 30 hours.

COMMON ERROR

ALERT

If the given values are not substituted into the proportion correctly, an error will occur. Write the equation for an indirect proportion first: $x_1y_1 = x_2y_2$. Then substitute the given values. Check the subscripts carefully as you substitute the values.

EXERCISES 11-5

Assume that the variables *x* and *y* are inversely proportional. Determine the missing value in each case.

1. If $x_1 = 7$, $x_2 = 4$, and $y_2 = 21$, determine the value of y_1.
2. If $x_1 = 3$, $y_1 = 12$, and $x_2 = 9$, determine the value of y_2.
3. If $y_1 = 1.5$, $x_2 = 2.5$, and $y_2 = 3$, determine the value of x_1.
4. If $x_1 = 25$, $x_2 = 5$, and $y_2 = 125$, determine the value of y_1.
5. If $x_1 = \frac{1}{2}$, $y_1 = 6$, and $y_2 = 9$, determine the value of x_2.
6. If $y_1 = 1$, $x_2 = 4$, and $y_2 = 7$, determine the value of x_1.